

LOGIC EXERCISES: WEEK 7

QUESTION 1

Construct proofs in natural deduction to show that the following L_1 sequents are correct. The first five are easy. Ish. The last three are harder.

- i. $(\neg Q \vee P), (P \rightarrow R) \vdash (Q \rightarrow R)$
- ii. $((P \vee Q) \rightarrow R) \vdash (P \rightarrow R)$
- iii. $(P \rightarrow R) \vdash ((P \wedge Q) \rightarrow R)$
- iv. $\vdash ((\neg P \rightarrow P) \rightarrow P)$
- v. $\neg P, (P \vee Q) \vdash Q$
- vi. $(P \rightarrow Q) \vdash (\neg P \vee Q)$
- vii. $(\neg P \vee Q) \vdash (P \rightarrow Q)$
- viii. $(P \leftrightarrow Q) \vdash ((P \wedge Q) \vee (\neg P \wedge \neg Q))$

QUESTION 2

Construct proofs in natural deduction to show that the following L_2 sequents are correct.

- i. $\forall x(Px \rightarrow Qx), \neg Qa \vdash \neg Pa$
- ii. $\forall x(Px \rightarrow Qx), Pa \vdash \exists xQx$
- iii. $\exists x\neg Px \vdash \neg \forall xPx$
- iv. $\neg \forall xPx \vdash \exists x\neg Px$
- v. $\exists x\neg Pxa \vdash \exists x\neg \forall yPyx$
- vi. $\exists y \forall x Rxy \vdash \forall x \exists y Rxy$
- vii. $\exists x \forall y (Rxy \leftrightarrow \neg Ryy) \vdash P$

QUESTION 3

Produce L_2 -structures that are counterexamples to the following incorrect claims of validity.

- i. $Pa, \forall x(Px \rightarrow Qx) \models \forall xQx$
- ii. $\forall xRxx, \forall x \forall y (Rxy \rightarrow Ryx) \models \forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz)$
- iii. $(\forall xPx \leftrightarrow \forall xQx) \models \forall x(Px \leftrightarrow Qx)$
- iv. $\forall x((Px \wedge Qx) \rightarrow Rax) \models \exists x(Rax \vee (Px \vee Qx))$
- v. $(\forall x \forall y Fxy \rightarrow \forall x \forall y Gxy) \models \forall x \forall y (Fxy \rightarrow Gxy)$
- vi. $(\exists x Fxa \rightarrow \exists x \exists y Gxy) \models \forall x \forall y (Fxa \rightarrow Gxy)$
- vii. $\forall x \exists y Rxy, \exists x \forall y Rxy \models \exists x \forall y Ryx$

QUESTION 4

Recall the following ambiguous English sentences from week 1, question 4. Can you disambiguate them in L_2 ? If so, give distinct formalisations for the different interpretations; if not, briefly explain why not.

- i. One morning I shot an elephant in my pyjamas.
- ii. This is a pretty little girl's dress.
- iii. Cherie's sister sleeps with her husband.
- iv. I hit my brother.
- v. Everyone at Lincoln loves someone at Hertford.
- vi. People think I'm taller than I am.
- vii. I want to buy a book.
- viii. Someone always spoils the fun.